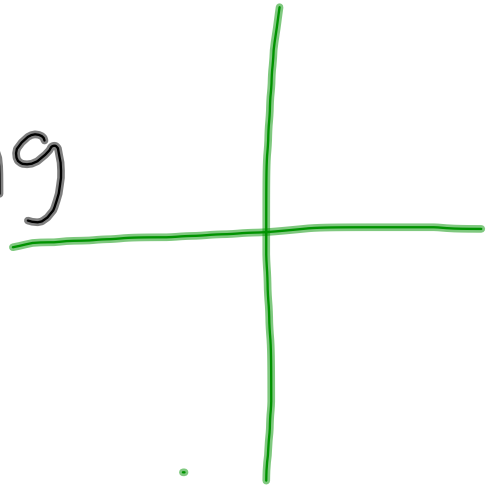
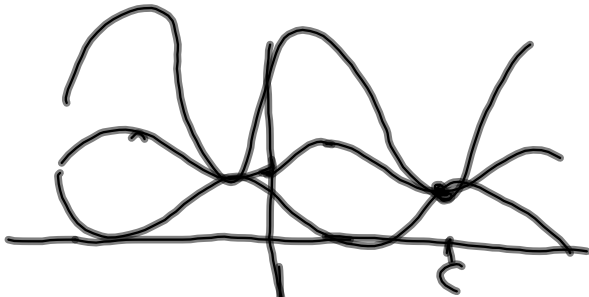
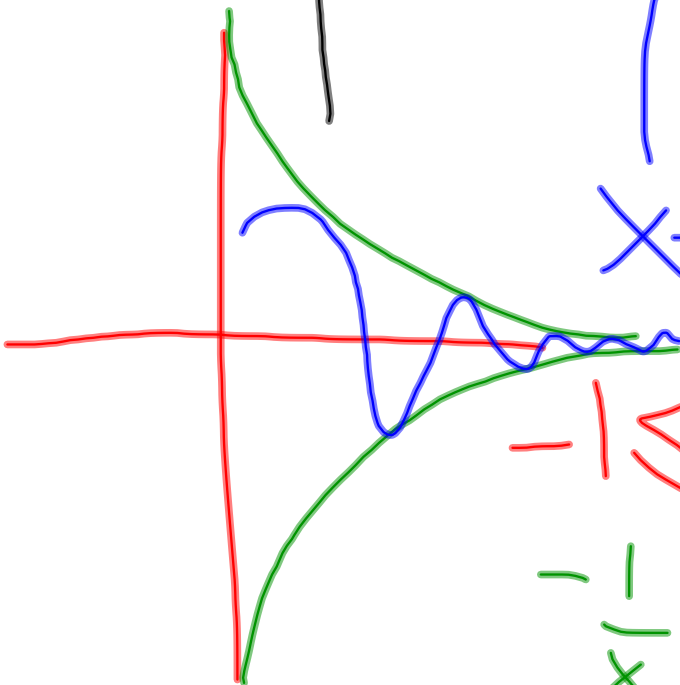


$$\lim_{x \rightarrow c} \sin x = \sin c$$

Squeezing



$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$



$$-1 \leq \sin x \leq 1$$

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\begin{aligned}
 & \lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{\sin x}{x} \right) \left(\frac{1}{x} \right) \\
 &= \infty
 \end{aligned}$$

The diagram shows the limit calculation with green annotations. A green arrow points from the $\sin x$ term in the second line to the $\frac{\sin x}{x}$ term, with a label $\rightarrow 1$. Another green arrow points from the $\frac{1}{x}$ term in the second line to the $\frac{1}{x}$ term in the third line, with a label $\rightarrow +\infty$.

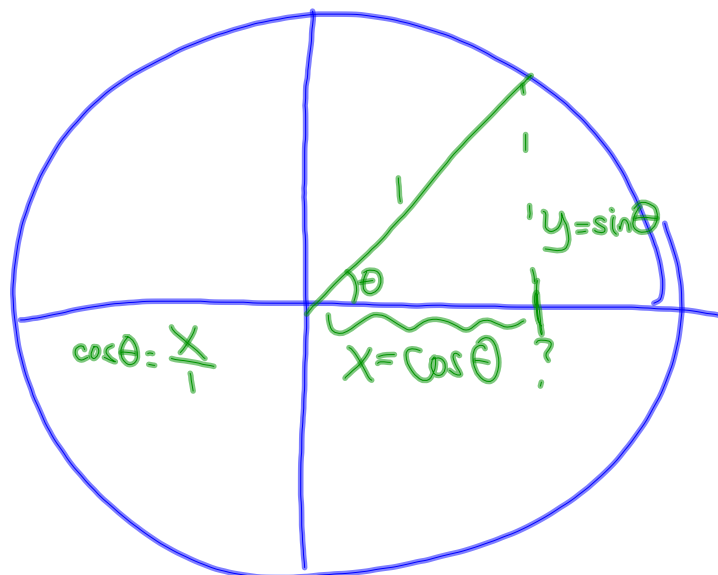
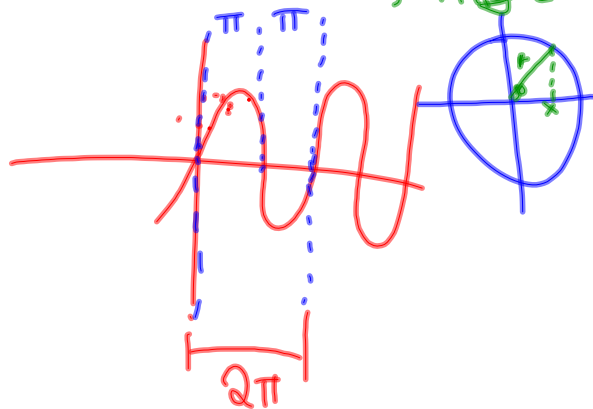
$$1. f(x) = \sin(x^2 - 2)$$

NO discontinuities

$$2. f(x) = \cos\left(\frac{x}{x-\pi}\right)$$

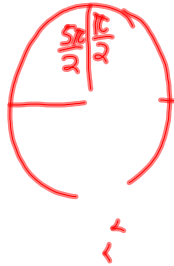
$$3. f(x) = \cot x = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

discontinuous at $x = n\pi$
 n , an integer



$$4.) f(x) = \sec(x) = \frac{1}{\cos(x)}$$

$$x \neq \frac{\pi}{2} \pm n\pi$$



$$5.) f(x) = \csc(x) = \frac{1}{\sin(x)}$$

$$\sin(x) = 0 = \sin(\pi)$$

$$x \neq \pi \notin 0 \notin \pi n$$

$$6.) f(x) = \frac{1}{1 + \sin^2(x)}$$

no discontinuity

$$1 + \sin^2(x) = 0$$

$$\sin^2(x) = -1$$

$$6^*) f(x) = \frac{1}{\sin^2(x) - 1}$$

$$x \neq \pi \pm 2\pi n$$

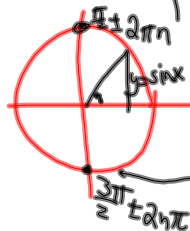
$$\sin^2(x) = 1$$

$$\sin(x) = 1$$

$$\sin(x) = -1$$

$$(\sin(x))^2 = 1$$

$$(\sin(x)) = \pm 1$$



$$\sin x = +1$$

$$\sin x = -1$$

$$\frac{3\pi}{2} = \frac{\pi}{2} + \pi$$

7) $f(x) = |\cos(x)|$

Always continuous



Start here ...

$f(x) = \sqrt{2 + \tan^2(x)}$

$\therefore 2 + \tan^2(x) \geq 0$

$\tan^2 x \geq 0$
 $\tan^2 x \geq 2$

$\tan^2 x = \left(\frac{\sin x}{\cos x} \right)^2$

$\cos x = 0$
 $x = \frac{\pi}{2} + n\pi$

no discontinuities arise when $2 + \tan^2 x < 0$

discontinuities

$$\sqrt{2 + \tan^2 x}$$

is $\tan x$ always defined?

No
not at

$$x = \frac{\pi}{2} \pm n\pi$$

when $\tan x$ is defined
do I ever take the square root of a negative number?

NO

$$2 + \tan^2 x \geq 2$$

13. $\lim_{x \rightarrow +\infty} \cos\left(\frac{1}{x}\right)$

$$= \cos\left(\lim_{x \rightarrow \infty} \frac{1}{x}\right) = \cos(0) = 1$$

